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CS 225

Asn 3.2: Set Operations

Section 2.2 (7th edition) = {2,4,12,16,18,20}

**2**) a) A ∩ B

b) A - B

c) A U B

d) U

**4**) Let A = {a,b,c,d,e} and B = {a,b,c,d,e,f,g,h}

a) A U B = {a,b,c,d,e,f,g,h} b) A ∩ B = {a,b,c,d,e} c) A - B = Ø d) B - A = {f,g,h}

**12**) Prove A U ( A ∩ B ) = A

This identity can be proven by being able to show that each side of the solution is the subset of the other.

We start off by trying to show that A U ( A ∩ B ) is a subset of A. If x € A U ( A ∩ B ), then (x € A) or ( x € A ∩ B ) by the law of unions. With the definition of intersections, we get (x € A) OR ( x € A and x € B). Since the element x is in both cases of A no matter what, then we know that A U ( A ∩ B ) is a subset of A.

Now to show that A is a subset of A U ( A ∩ B ), let x € A. Then, (x € A) or ( x € A and x € B). This means that x € A U ( A ∩ B), by the definition of unions. This shows that subset A has an element x, that is also in A U ( A ∩ B ), which means A is a subset of A U ( A ∩ B ).

**16)** a) Show ( A ∩ B) ⊆ A

let x € (A ∩ B). this also means x € A and x € B using the definition of interections. If x € A, that means ( A ∩ B) is a subset of A.

b) A ⊆ (A U b)

let x € A. By the definition of unions, x € A or x € B. so x € A U B, which shows that A is a subset of A U B

c) A - B ⊆ A

let x € A - B. This is a difference of A and B, which can be written as x € A and x ~~€~~  B. That mean x € A , so A - B must be a subset of A.

d) A ∩ ( B - A ) = Ø

The approach here is to use contradiction. We can assume that A ∩ ( B - A ) is not Ø, and that there is an element x that such that x € A, x € B and x ~~€ A~~, By the definition of intersection and differences. But that is a contradiction since cannot be x € A and x ~~€ A~~ . So our proof for contradiction was false, which means A ∩ ( B - A ) = Ø is true.

e) A U ( B - A ) = A U B

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| A | B | B - A | A U (B - A ) | A U B |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 |

The proof is complete, The two collumns are the same.

18) a) show ( A U B) ⊆ ( A U B U C )

let x € ( A U B). Then x € A or x € B, by definition of union. Then that means for

( A U B U C), x € A, x € B, or x € C. We can show that x € ( A U B U C) by using union definition, which means that ( A U B) ⊆ ( A U B U C ).

b) show ( A ∩ B ∩ C) ⊆ ( A ∩ B)

let x € (A ∩ B ∩ C). We can distribute this out by the law of intersection, x € A, x € B, and

x € C. Using this same method for ( A ∩ B), we get x € A, and x € B. It doesn't matter that x € C, because as long as x € A, x € B is true in ( A ∩ B ∩ C), then x € (A ∩ B) as well. Which concludes our proof that ( A ∩ B ∩ C) ⊆ ( A ∩ B)

c) show (A - B) - C ⊆ (A - C)

let x € (A-B) - C). The definition of difference allows us to write € A, x € ~~B,~~ and x ~~€ C, .~~ For the (A ∩ B), we use the same rules to show that x € A and x ~~€ C,~~ This completes our proof since we have shown that x is an element in A, but not C on both accounts (A - B) - C ⊆ (A - C).

d) ( A - C ) ∩ (C - B) = Ø

We prove this by contradiction. Let's assume that ( A - C ) ∩ (C - B) contains an x that the case is x € (A - C) ∩ (C 0 B). This means x € (A - C) and x € (C-B ) by definition of intersection. Then by the definition of differences, x € A, and x is not € C, and x € C, and x is not € B. x cannot both be in C and NOT be in C at the same time, which is a contradiction. Which proves our original hypothesis ( A - C ) ∩ (C - B) = Ø is true.

e. (B - A) U ( C - A ) = ( B U C) - A

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| A | B | C | C - A | B - A | B U C | (B - A) U (C- A) | ( B U C) - A |
| 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

20) Show ( A ∩ B) U (A ∩ ) = A

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| A | B | ( A ∩ B) |  | (A ∩ ) | ( A ∩ B) U (A ∩ ) |
| 1 | 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 |